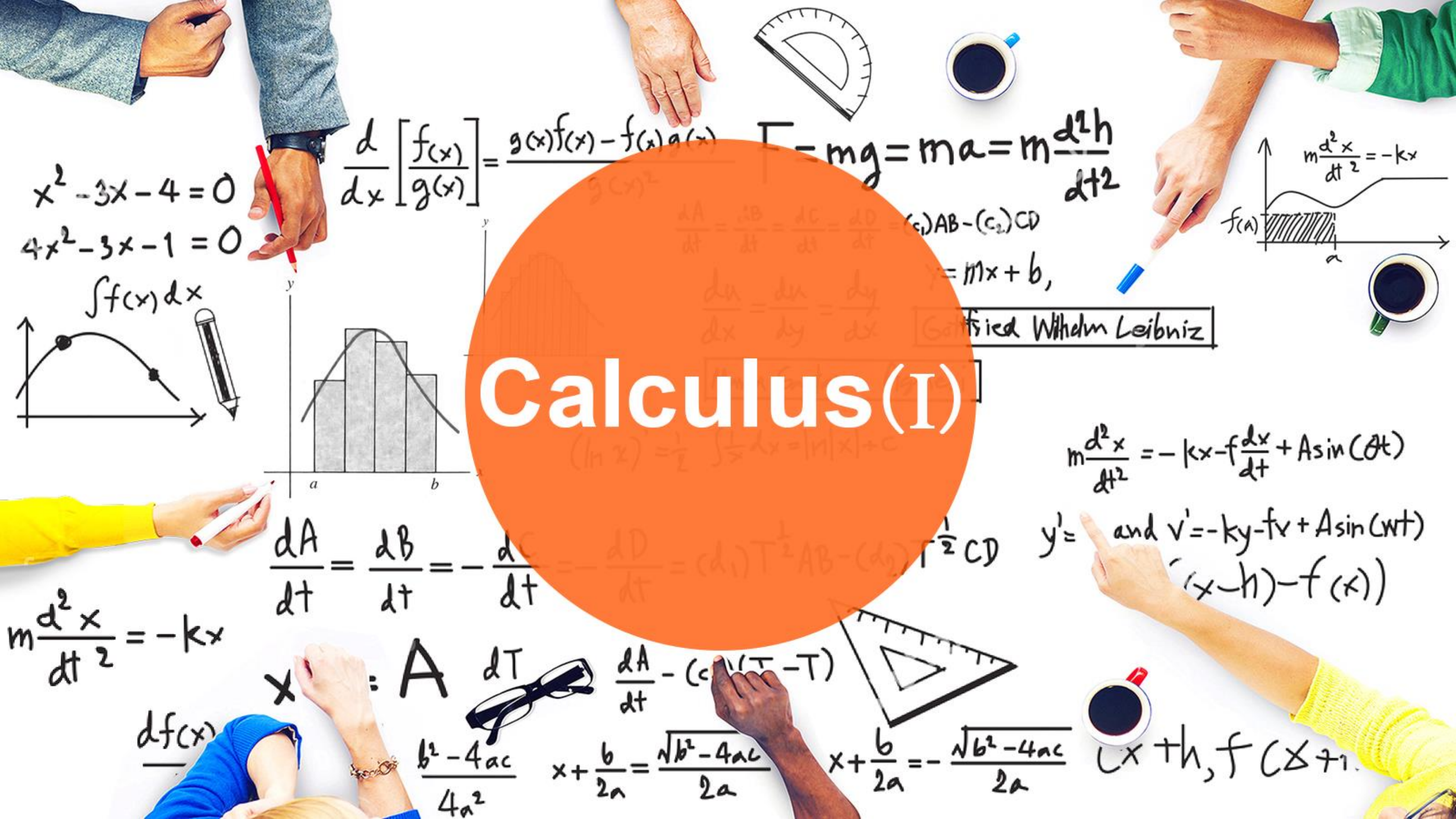
# Calculus(I)



# 2.4 Derivatives of Trigonometric Functions

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# How to find the derivatives of the following functions?

$$(\sin x)' = ? \qquad (\cos x)' = ?$$

$$(\tan x)' = ? \qquad (\cot x)' = ?$$

According to the definition of derivative.

# Theorems

Th A

The functions $f(x) = \sin x$ and $g(x) = \cos x$ are both differentiable and,

$$D_x(\sin x) = \cos x\,, D_x(\cos x) = -\sin x$$

Th B

For all points $x$ in the function's domain,

$$D_x \tan x = \sec^2 x\,, D_x \cot x = -\csc^2 x$$

$$D_x \sec x = \sec x \tan x\,, D_x \csc x = -\csc x \cot x$$

# Example 1

Find $y'$ if $y = x^n, n$ is positive integer.

$$(x^n)' = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0}[nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \cdots + h^{n-1}] = nx^{n-1}$$

Namely, $(x^n)' = nx^{n-1}$

Generally, $(x^\mu)' = \mu x^{\mu-1}.$ $(\mu \in R)$

Eg. $(\sqrt{x})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

$(x^{-1})' = (-1)x^{-1-1} = -\frac{1}{x^2}$

# Example 2

Find $f'(x)$ if $f(x) = a^x$ $(a > 0, a \neq 1)$.

$$(a^x)' = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$

$$= a^x \ln a.$$

Namely, $(a^x)' = a^x \ln a$

$$(e^x)' = e^x.$$

# Example 3

Find $y'$ if $y = \log_a x \ (a > 0, a \neq 1)$.

$$y' = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h}$$

$$= \lim_{h \to 0} \frac{\log_a(1 + \frac{h}{x})}{\frac{h}{x}} \cdot \frac{1}{x}$$

$$= \frac{1}{x} \lim_{h \to 0} \log_a (1 + \frac{h}{x})^{\frac{x}{h}} = \frac{1}{x} \log_a e.$$

Namely, $(\log_a x)' = \frac{1}{x} \log_a e \qquad (\ln x)' = \frac{1}{x}.$

# Example 4

$$[f^{-1}(x)]' = \frac{1}{f'(y)}$$

Find the derivative of $y = \sin^{-1} x$.

Original function Inverse function

$\because x = \sin y$ is monotonic and differentiable when $I_y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $(\sin y)' = \cos y$ **> 0,** $\therefore$ when $I_x \in (-1,1)$

$$(\arcsin x)' = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

In the similar way, $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}.$

$$(\arctan x)' = \frac{1}{1+x^2}; \quad (\operatorname{arc}\cot x)' = -\frac{1}{1+x^2}.$$

# The propety of Trigonometric Functions

According to the propety of Trigonometric Functions:

$$\arccos x = \frac{\pi}{2} - \arcsin x, \quad \Rightarrow \quad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}},$$

$$\operatorname{arccot} x = \frac{\pi}{2} - \arctan x, \quad \Rightarrow \quad (arc\cot x)' = -\frac{1}{1+x^2}.$$

# The Derivative Formulas

$$(C)' = 0$$

$$(x^{\mu})' = \mu x^{\mu-1}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\csc x)' = -\csc x \cot x$$

$$(a^x)' = a^x \ln a$$

$$(e^x)' = e^x$$

$$(\log_a x)' = \frac{1}{x \ln a}$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

# Derivatives of Trigonometric Functions